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Design of Coupled Microstrip Lines by Optimization Methods

STANISŁAW ROSŁONIEC

Abstract —**The design of two coupled microstrip lines is converted into an optimization problem, which is then solved by two methods of nonlinear mathematical programming. New formulas for calculating the first approximation of the solution, i.e., the starting point of the optimization, are given.**

I. Introduction

The design of two parallel coupled microstrip lines has been extensively studied in the literature (see e.g. [1]–[9]). The formulas given in [7] and [8] are most useful for accuracy reasons. They make it possible to calculate the line impedances Z_{0e} and Z_{0o} provided the geometric dimensions and the permittivity of the dielectric substrate are given. In practice, the reverse problem is usually solved, which is equivalent to the problem of solving two nonlinear equations with strip width and slot width as variables. An algorithm for solving this set of equations should be reliable, accurate, and quickly convergent. In this paper the design problem of two coupled microstrip lines is converted into an optimization problem, which is then solved by two methods of nonlinear mathematical programming, applied subsequently when approaching the solution. New formulas for calculating the first approximation of the solution being sought, i.e., the starting point for the optimization, are given. Properties of the design algorithm are illustrated by calculated results.

II. The Design Algorithm

A transverse section of the coupled lines being considered is shown in Fig. 1. If $u = W/h$ and $g = S/h$, the characteristic impedances Z_{0e} and Z_{0o} of these lines can be expressed as

$$Z_{0e} = F_1(u, g, \epsilon_r)$$
$$Z_{0o} = F_2(u, g, \epsilon_r) \tag{1}$$

where F_1 and F_2 are relations given in the papers cited above, e.g., [7]. The problem of designing two coupled microstrip lines for given impedances Z_{0e}, Z_{0o} and permittivity ϵ_r of a dielectric substrate consists of determination of the values u_S and g_S

$$F_1(u_S, g_S, \epsilon_r) - Z_{0e} = 0$$
$$F_2(u_S, g_S, \epsilon_r) - Z_{0o} = 0. \tag{2}$$

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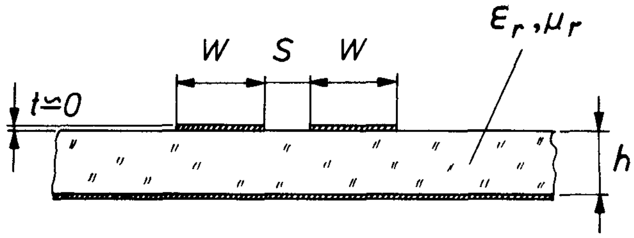

Fig. 1. Transverse section of the coupled microstrip lines.

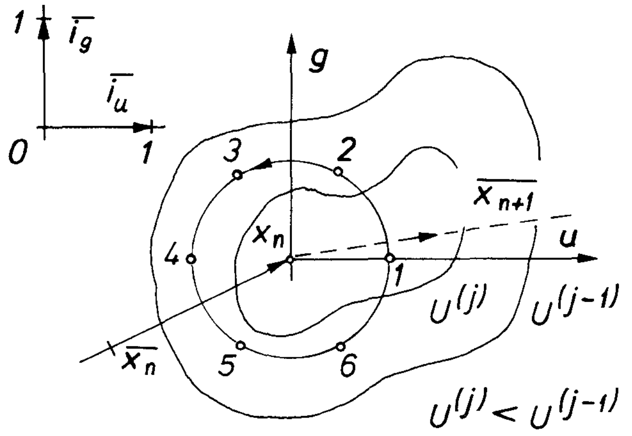

Fig. 2. Diagram of the search process.

It can easily be seen that solution of (2) is equivalent to finding a global minimum of the following function:

$$U(u, g, \epsilon_r) = [F_1(u, g, \epsilon_r) - Z_{0e}]^2 + [F_2(u, g, \epsilon_r) - Z_{0o}]^2. \tag{3}$$

From (2) it is evident that function (3) reaches its global minimum equal to zero at the point being sought (u_S, g_S). According to (3) the problem being considered can be written in the form

$$\min_{(u,g) \subset D} U(u, g, \epsilon_r) \tag{4}$$

where D is the set of values of u and g possible from the point of view of the construction. The above-formulated minimization problem has been solved by various methods of nonlinear mathematical programming, i.e., the steepest descent, Fletcher–Reeves, and Davidon–Fletcher–Powell methods [10].

From the performed analysis it is seen that the best results, in the sense of the criterion given earlier, are ensured by the Davidon–Fletcher–Powell method and the direct nongradient search. In the prepared subroutine, entitled "coupled lines" (see next page), the direct search is conducted at six points lying on a circle (Fig. 2) with radius $h_n/2$ [9]. Function (3) takes its minimal value, while searching in direction $\bar{x}_n$ with the step h_n, at point x_n, which is the center of this circle. This nongradient method is additionally used when $U(u, g, \epsilon_r) < 3$. The search terminates if $U(u, g, \epsilon_r) < Z_{0e} \cdot Z_{0o}/10000$, which ensures a relative approximation accuracy not worse than 1 percent for the impedances.

The reasonable choice of the first approximation, i.e., the starting point for optimization, significantly affects the computation time. In the coupled lines subroutine, the first approximation is found according to the formula [9]

$$u_0 = |F_3(Z_0, \epsilon_r) \cdot F_4(k)|$$
$$g_0 = |F_3(Z_0, \epsilon_r) \cdot F_5(k, \epsilon_r)| \tag{5}$$

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TABLE III

COUPLED LINES SUBROUTINE (IN BASIC)

```
10 REM CL
20 PRINT "COUPLED LINES"
30 PRINT
40 PRINT "by S.Rosloniec, Warsaw, 1987"
50 PRINT
60 PRINT "Data:"
70 PRINT
80 PRINT "Zoe=";: INPUT zoe;: PRINT zoe,"ohm"
90 PRINT "Zoo=";: INPUT zoo;: PRINT zoo,"ohm"
100 PRINT "er =";: INPUT er;: PRINT er
110 PRINT "ur =1"
120 PRINT
130 IF zoe<=zoo THEN PRINT "Error:Zoe<=Zoo": GO TO 0050
140 IF er<1 THEN PRINT "Error:er<1": GO TO 0050
150 PRINT "Wait please !"
160 PRINT
170 DIM a(6)
180 DIM b(6)
190 DIM c(6)
200 FOR i=1 TO 6
210 READ a(i)
220 READ b(i)
230 READ c(i)
240 NEXT i
250 DATA 1,.02,.015,-.300927,-.622893,-2.451164,3.209206,17.192310,50.561737,-2
7.282781,-68.946372,-260.2209,56.609340,104.740790,536.73042,-37.746369,-16.1482
74,-365.30851
260 LET zo=SQR (zoe*zoo)
270 LET k=(zoe-zoo)/(zoe+zoo)
280 LET aw=EXP (zo/42.4*SQR (er+1))-1
290 LET ww=8/aw*SQR (aw/11*(7+4/er)+1/0.81*(1+1/er))
300 LET f2=0
310 LET f3=0
320 FOR i=1 TO 6
330 LET f2=f2+a(i)*k↑(i-1)
340 NEXT i
350 IF k>=.5 THEN LET kk=.5: GO TO 0370
360 LET kk=k
370 FOR i=1 TO 6
380 LET f3=f3+(b(i)-c(i)*(9.6-er)/7.05)*(.6-kk)↑(i-1)
390 NEXT i
400 IF k>=.5 THEN LET f3=f3*(1-k)/.5
410 LET w=ABS (ww*f2)
420 LET s=ABS (ww*f3)
430 GO SUB 0510
440 PRINT "Results:"
450 PRINT
460 PRINT "W/h=";w
470 PRINT "S/h=";s
480 PRINT "efe=";efe
490 PRINT "efo=";efo
500 STOP
510 REM Procedure CL
520 LET it=0
530 LET wo=w
540 LET so=s
550 LET h=.1*(1-INT (10*k)/10)
560 GO SUB 1360
570 LET f=ze
580 LET g=zo
590 LET w=w+h
600 GO SUB 1360
610 LET f1=ze
620 LET g1=zo
630 LET w=w-h
640 LET s=s+h
650 GO SUB 1360
660 LET f2=ze
670 LET g2=zo
680 LET s=s-h
690 LET pw1=2*(f*(f1-f)+g*(g1-g))/h
700 LET ps1=2*(f*(f2-f)+g*(g2-g))/h
710 LET mo1=SQR (pw1*pw1+ps1*ps1)+1e-12
720 IF (f*f+g*g)<=zoe*zoo/10000 THEN RETURN
730 IF (f*f+g*g)>3 THEN GO TO 0810
740 LET pr=h/2
750 FOR t=1 TO 6
760 LET w=ABS (wo+pr*COS (PI/3*t))
770 LET s=ABS (so+pr*SIN (PI/3*t))
780 GO SUB 1360
790 IF (ze*ze+zo*zo)<=zoe*zoo/10000 THEN RETURN
800 NEXT t
810 IF it>=1 THEN GO TO 0860
820 LET d1=1
830 LET d2=0
840 LET d3=0
850 LET d4=1
860 LET hh=h
870 LET w=ABS (wo-hh*(pw1*d1+ps1*d2)/mo1)
880 LET s=ABS (so-hh*(pw1*d3+ps1*d4)/mo1)
890 GO SUB 1360
900 IF (f*f+g*g)<=zoe*zoo/10000 THEN RETURN
910 IF (ze*ze+zo*zo)>=(f*f+g*g) OR hh>3 THEN GO TO 0960
920 LET f=ze
930 LET g=zo
940 LET hh=hh+h
950 GO TO 0870
960 LET w=ABS (wo-(hh-h/2)*(pw1*d1+ps1*d2)/mo1)
970 LET s=ABS (so-(hh-h/2)*(pw1*d3+ps1*d4)/mo1)
980 LET dw=w-wo
990 LET wo=w
1000 LET ds=s-so
1010 LET so=s
1020 IF h<=0.001 THEN LET h=(1.25-.05*it)*h: GO TO 1040
1030 LET h=.7*h
1040 GO SUB 1360
1050 LET f=ze
1060 LET g=zo
1070 LET w=w+h
1080 GO SUB 1360
1090 LET f1=ze
1100 LET g1=zo
1110 LET w=w-h
1120 LET s=s+h
1130 GO SUB 1360
1140 LET f2=ze
1150 LET g2=zo
1160 LET s=s-h
1170 LET pw2=2*(f*(f1-f)+g*(g1-g))/h
1180 LET ps2=2*(f*(f2-f)+g*(g2-g))/h
1190 LET mo2=SQR (pw2*pw2+ps2*ps2)+1e-12
1200 LET dpw=pw2-pw1+1e-12
1210 LET dps=ps2-ps1+1e-12
1220 LET k1=dw*dpw+ds*dps+1e-12
1230 LET k2=dpw*(d1*dpw+d2*dps)+dps*(d3*dpw+d4*dps)+1e-12
1240 LET k3=dpw*d1+dps*d3
1250 LET k4=dpw*d2+dps*d4
1260 LET d1=d1+dw*dw/k1-(d1*dpw*k3+d2*dps*k3)/k2
1270 LET d2=d2+dw*ds/k1-(d1*dpw*k4+d2*dps*k4)/k2
1280 LET d3=d3+ds*dw/k1-(d3*dpw*k3+d4*dps*k3)/k2
1290 LET d4=d4+ds*ds/k1-(d3*dpw*k4+d4*dps*k4)/k2
1300 LET it=it+1
1310 LET pw1=pw2
1320 LET ps1=ps2
1330 LET mo1=mo2
1340 IF it=5 THEN LET it=0
1350 GO TO 0720
1360 REM Subroutine 1360
1370 LET a=1+1/49*LN ((w↑4+(w/52)↑2)/(w↑4+0.432))+1/18.7*LN (1+(w/18.1)↑3)
1380 LET b=0.564*((er-0.9)/(er+3))↑0.053
1390 LET c=(EXP (s↑0.68)+EXP (-s↑0.68))/2
1400 LET d=s*EXP (-s)+w*(20+s*s)/(10+s*s)
1410 LET e=1+s/1.45+(s↑2.09)/3.95
1420 LET i=0.2306+1/301.8*LN (s↑10/(1+(s/3.77)↑10))+1/5.3*LN (1+0.646*s↑1.175)
1430 LET j=EXP (-s)/2
1440 IF s<=1e-5 THEN LET s=1e-5
1450 LET m=0.2175+(4.113+(20.36/s)↑6)↑-0.251+1/323*LN (s↑10/(1+(s/13.8)↑10))
1460 LET n=(1/17.7+EXP (-6.424-0.76*LN s-(s/0.23)↑5))*LN ((10+68.3*s↑2)/(1+32.5*
s↑3.093))
1470 LET o=1.729+1.175*LN (1+0.627/(s+0.327*s↑2.17))
1480 LET p=EXP (-0.745*s↑0.295)/c
1490 LET q=EXP (-1.366-s)
1500 LET r=1+0.15*(1-EXP (1-(er-1)*(er-1)/8.2)/(1+s↑-6))
1510 LET v=0.8645*w↑0.172
1520 LET ve=v/(e*(j*w↑m+(1-j)*w↑-m))
1530 LET vo=ve-o/e*EXP (i*w↑-n*LN w)
1540 LET fe=(1+10/d)↑(-a*b)
1550 LET f3=1-EXP (-0.179*s↑0.15-0.328*s↑r/(LN (EXP 1+(s/7)↑2.8)))
1560 LET f4=f3*EXP (p*LN w+q*SIN (PI*LN w/LN 10))
1570 LET f5=6+(2*PI-6)*EXP -(30.666/w)↑0.7528
1580 LET fo=f4*(1+10/w)↑(-a*b)
1590 LET efe=(er+1)/2+(er-1)/2*fe
1600 LET efo=(er+1)/2+(er-1)/2*fo
1610 LET zo1=60*LN (f5/w+SQR (1+(2/w)↑2))
1620 LET ze=zo1/(SQR (efe)*(1-zo1*ve/(120*PI)))-zoe
1630 LET zo=zo1/(SQR (efo)*(1-zo1*vo/(120*PI)))-zoo
1640 RETURN
```

where

$$Z_0=\sqrt{Z_{0e}Z_{0o}}$$

$$k=\frac{Z_{0e}-Z_{0o}}{Z_{0e}+Z_{0o}}$$

$$F_3=\frac{8}{A}\sqrt{\frac{A}{11}\left(7+\frac{4}{\epsilon_r}\right)+\frac{1}{0.81}\left(1+\frac{1}{\epsilon_r}\right)}$$

$$A=\exp\left(\frac{Z_0}{42.4}\sqrt{\epsilon_r+1}\right)-1$$

$$F_4=\sum_{i=1}^{6}a_i k^{(i-1)}$$

$$F_5=\begin{cases}\displaystyle\sum_{i=1}^{6}\left[b_i-c_i\left(\frac{9.6-\epsilon_r}{7.05}\right)\right](0.6-k)^{(i-1)} & \text{for } k\leqslant 0.5\\[2ex] \left[F_5(k=0.5,\epsilon_r)\right]\left(\dfrac{1-k}{0.5}\right) & \text{for } k>0.5.\end{cases}$$

The values of the coefficients a_i, b_i, and c_i, where $i=1,2,\cdots,6$, of functions F_4 and F_5 are given in Table I. The function F_3, formulated according to the Wheeler formula, is used in relations

TABLE I

i	a_i	b_i	c_i
1	1	0.020	0.015
2	-0.501	-0.623	-2.451
3	3.209	17.192	50.561
4	-27.282	-68.946	-260.220
5	56.609	104.740	536.740
6	-37.746	-16.148	-365.308

TABLE II

ε_r = 2.55		Starting point		Solution	
Z_{0e}, ohm	Z_{0o}, ohm	u_0	g_0	u_S	g_S
51.60	48.44	2.752	2.307	2.802	2.307
55.23	45.23	2.719	1.110	2.743	1.152
62.79	39.82	2.518	0.366	2.538	0.332
69.37	36.04	2.245	0.135	2.310	0.126
86.74	28.82	1.660	0.015	1.743	0.011

TABLE III

ε_r = 9.60		Starting point		Solution	
Z_{0e}, ohm	Z_{0o}, ohm	u_0	g_0	u_S	g_S
51.60	48.44	0.988	2.521	0.994	2.658
55.23	45.23	0.976	1.425	0.980	1.352
62.78	39.82	0.947	0.425	0.967	0.390
69.37	36.04	0.810	0.300	0.827	0.296
86.74	28.82	0.595	0.071	0.618	0.070

(5) [3]. After the starting point (u_0, g_0) has been assessed, the search for the minimum of function (3) is realized with initial step length $h = 0.1$, which is subsequently reduced to $h_n = 0.7h_{n-1}$ for every subsequent search direction $\overline{x_n}$ (Fig. 2). Further reduction is not performed if h_n is less than 0.001.

The values of u and g are always positive, so they are calculated from the formula [9]

$$u_{n+1} = \left| u_n - h_n \frac{\overline{x_n} \cdot \overline{i_u}}{|\overline{x_n}| + \epsilon} \right|$$

$$g_{n+1} = \left| g_n - h_n \frac{\overline{x_n} \cdot \overline{i_g}}{|\overline{x_n}| + \epsilon} \right| \quad (6)$$

where $\overline{i_u}$ and $\overline{i_g}$ are versors and ϵ is a small positive number. Owing to the number ϵ, equal to, e.g., 10^{-12}, the computation algorithm is protected from error due to dividing by zero.

III. Calculated Results

It has been found from numerical experiment that the algorithm presented here satisfies very well the initially established requirements for $0.03 \leqslant k \leqslant 0.6$ and $2 \leqslant \epsilon_r \leqslant 12$. The solutions were found in most cases after a few iterations, proving that the applied methods are efficient. The quality of the formulas for calculating the first approximation is well illustrated by the results given in Tables II and III. In these tables, the values of the effective line permittivities for the even and odd excitations are not given because they are related to the solution (u_S, g_S) by explicit formulas and can be calculated without difficulty.

IV. Conclusions

The formulas given in [7] and [8] make it possible to assess the impedance Z_{0e} and Z_{0o} of two parallel coupled microstrip lines provided their geometric dimensions are given. In practice, the reverse problem is usually solved, where geometric dimensions are evaluated for given impedances Z_{0e} and Z_{0o}. In this paper an algorithm for solving this problem is presented. The basis of this algorithm consists in converting the problem of solving two nonlinear equations into an equivalent minimization problem which is then solved by using the Davidon–Fletcher–Powell and direct, i.e. nongradient, search methods. Formulas for calculating the initial point are given.

For $\sqrt{Z_{0e} Z_{0o}} = 50\ \Omega$, $2 \leqslant \epsilon_r \leqslant 12$, and $0.03 \leqslant k \leqslant 0.6$, this point is close to the solution being sought. It was found that in many cases the search for the solution can successfully be concluded due to an additional nongradient search on the circle, which makes the presented algorithm more reliable and effective.

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